Es.1: Det. C' e raggio di $\gamma: \begin{cases} x^2+y^2+z^2+2x-6y+4z-6=0 \\ 2x+y-z-9=0 \quad : \pi \end{cases}$

e l'eq. della retta tg a γ in $P=(3,3,0)$.

• S: $C=(-1,+3,-2)$

$\text{raggio}_S = \frac{1}{2}\sqrt{80} = 2\sqrt{5}$

$d(C,C') = d(C,\pi) = \frac{|-2+3+2-9|}{\sqrt{4+1+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}$

$\text{raggio}_\gamma = \sqrt{(2\sqrt{5})^2-(\sqrt{6})^2} = \sqrt{14}$

$C': \rho\begin{pmatrix} x+1 & y-3 & z+2 \\ 2 & 1 & -1 \end{pmatrix} = 1$

$\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1} \qquad r: \begin{cases} y-3+z+2=0 \\ x+1+2z+4=0 \end{cases}$

$C' = r\cap\pi \quad \begin{cases} y+z-1=0 \\ x+2z+5=0 \\ 2x+y-z-9=0 \end{cases} \quad \dots \begin{cases} y=4 \\ x=1 \\ z=-3 \end{cases} \quad C'=(1,4,-3)$

• eq. del piano tangente a S in P : $\perp \overrightarrow{CP} = (3+1, 3-3, 0+2)$
$= (4, 0, 2)$

$\exists!\ 4x + 2z + k = 0$

per P : $12 + k = 0 \quad k = -12$

• $4x + 2z - 12 = 0$

$$r : \begin{cases} 2x + y - z - 9 = 0 \\ 4x + 2z - 12 = 0 \end{cases}$$

ES.2: Det. le eq. cartesiane della circonf. che passa per $A = (1,2,0)$, $B = (0,1,0)$ $D = (4,0,-1)$.

π : $A \in \pi, B \in \pi, D \in \pi$

$\downarrow$

$$\bullet\ \pi : \begin{vmatrix} x-1 & y-2 & z-0 \\ 0-1 & 1-2 & 0-0 \\ 4-1 & 0-2 & -1-0 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} x-1 & y-2 & z \\ -1 & -1 & 0 \\ 3 & -2 & -1 \end{vmatrix} = 0$$

π : $x - y + 5z + 1 = 0$

• piani assiali: α_{AB}:

$$(x-1)^2+(y-2)^2+(z-0)^2=(x-0)^2+(y-1)^2+(z-0)^2$$

$$\dots \quad x+y-2=0$$

$$\alpha_{BD}: (x-1)^2+(y-2)^2+z^2=(x-4)^2+(y-0)^2+(z+1)^2$$

$$\dots \quad 4x-y-z-8=0$$

$$C' = \pi \cap \alpha_{AB} \cap \alpha_{BD}$$

$\downarrow$

(centro della circonferenza)

$$C': \begin{cases} x-y+5z+1=0 \\ x+y-2=0 \\ 4x-y-z-8=0 \end{cases} \quad \dots \quad \begin{cases} y=\frac{1}{9} \\ x=\frac{17}{9} \\ z=-5/9 \end{cases}$$

$$C' = \left(\frac{17}{9}, \frac{1}{9}, -5/9\right)$$

$$\text{raggio} = d(C', A) = \dots = \frac{\sqrt{42}}{3}$$

$$\gamma: \begin{cases} \left(x-\frac{17}{9}\right)^2+\left(y-\frac{1}{9}\right)^2+\left(z+\frac{5}{9}\right)^2=\frac{42}{9} \\ x-y+5z+1=0 \end{cases}$$

ES 3: Det. l'eq. della circonf. γ che ha centro sulla retta $r: x-y+1 = z-x = 0$, $\perp$ al piano che contiene r e passa per $A=(2,-1,3)$. Si det. l'eq. della tg in A a γ.

- $r: \begin{cases} x-y+1=0 \\ z-x=0 \end{cases}$ par. π ($\perp r$): $\left[\left(\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}, -\begin{vmatrix} 1 & 0 \\ -1 & 1 \end{vmatrix}, \begin{vmatrix} 1 & -1 \\ -1 & 0 \end{vmatrix} \right)\right] = [(+1,+1,+1)]$

π: $\mathcal{F}_1$: $x+y+z+k=0$

A: $2-1+3+k=0 \Rightarrow k=-4$

- $\pi: x+y+z-4=0$

$C: \begin{cases} x-y+1=0 \\ z-x=0 \\ x+y+z-4=0 \end{cases} \cdots \begin{cases} y=2 \\ z=1 \\ x=1 \end{cases}$ $C=(1,2,1)$

raggio $= d(C,A) = \dots = \sqrt{14}$

$\gamma: \begin{cases} (x-1)^2+(y-2)^2+(z-1)^2=14 \\ x+y+z-4=0 \end{cases}$

tg: π: $\perp \vec{CA}$: $[(2-1, -1-2, 3-1)] = [(1,-3,2)]$

A: $x - 3y + 2z + k = 0$

$\hookrightarrow$ $2 + 3 + 6 + k = 0$ $k = -11$

$$\begin{cases} x - 3y + 2z - 11 = 0 \\ x + y + z - 4 = 0 \end{cases}.$$

Luogo geometrico in $E_3(\mathbb{R})$ ottenuto mediante rotazione di punti attorno ad una retta (asse)

ES.1: Si det. l'eq. cartesiana del luogo geom. $\mathcal{L}$ descritto da $P = (-1, -2, 3)$ attorno a

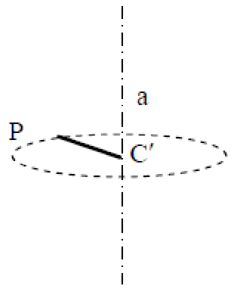
$a: \begin{cases} x = 0 \\ y = 3 \end{cases}$

• π_1: Pda $= [(\ldots \quad)] = [(0,0,1)]$

$\mathcal{F}_1$: $z + k = 0$

P: $-3 = +k$ $\rightarrow$ π: $z = 3$

$$C': \begin{cases} x=0 & a \\ y=3 & \\ z=3 & \pi \end{cases} \quad (0,3,3)$$

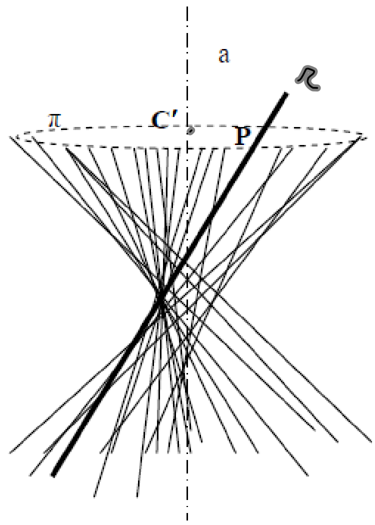


$$\text{raggio} = d(C', P) = \ldots = \sqrt{26}$$

$$\gamma: \begin{cases} x^2+(y-3)^2+(z-3)^2=26 \\ z=3 \end{cases}$$

Es 2: $a: \begin{cases} x+2y=0 \\ x+2y-z=0 \end{cases}$

$r: \begin{cases} z=1 \\ y=x \end{cases}$



- $\underset{\in r}{P} = (t, t, 1)$

$P_{da} = \ldots = [(-2,1,0)]$

$$\left.\begin{array}{l} \mathcal{F}_\perp: -2x+y+k=0 \\ P: -2t+t+k=0 \quad k=t \end{array}\right\} \pi: -2x+y+t=0$$

$$C': \begin{cases} x+2y=0 \\ x+2y-z=0 \\ -2x+y+t=0 \end{cases} \quad \therefore \quad \begin{cases} x = 2t/5 \\ z=0 \\ y = -t/5 \end{cases}$$

$$\text{raggio} = d(C',P) = \dots = \sqrt{\frac{9}{5}t^2+1}$$

$$\gamma: \begin{cases} -2x+y+t=0 \\ \left(x-\frac{2}{5}t\right)^2+\left(y+\frac{t}{5}\right)^2+z^2 = \frac{9}{5}t^2+1 \end{cases} \qquad t\in\mathbb{R}$$